1) The Reaction Wheel Pendulum (RWP)

- System characterization:
  - Nonlinear;
  - Underactuated (2 DOF, 1 actuator);
  - Upper equilibrium is unstable;
  - Fast dynamic challenges online optimization;
  - Control hardware: standard industrial PLC.
- Nonlinear tracking control problem:
  - Swingup from lower to upper equilibrium.

2) Tracking by Nonlinear Model Predictive Control (NMPC)

- NMPC: Optimization based method for nonlinear feedback control [1].
- Challenges:
  - The time needed to derive the solution of the optimization problem online introduces a delay between the measurement and the control action.
  - This can lead to destabilization or cause performance drawbacks.
- NMPC was not able to swing up the pendulum, due to long prediction horizon (T=2s), limited computation power and fast dynamics.

3) Tracking by 2-Degree-of-Freedom Control Structure

- Feedback control ensures asymptotic tracking of a reference trajectory.
- Performance depends on the quality of the reference trajectories.
- Offline: For a-priority known rest-to-rest transitions [2].
- Online: To react to user inputs - e.g. joystick commands.
- Goal of our approach: online trajectory optimization.

4) Proposal: Tracking Control by NMPC-"Model Control Loop" [3]

- Model Control Loop (MCL) - simulate a closed loop online:
  - Use simulated signals as reference- and feedforward-control trajectories for a nonlinear trajectory tracking controller applied to the plant.
- Combine advantages of NMPC and 2-degrees-of-freedom control:
  - Employ NMPC algorithms for online optimization of system trajectories;
  - Resulting optimal state- and control-trajectories are used as reference.
- Avoid the challenges of applying NMPC to a real plant:
  - No model uncertainty in NMPC-loop;
  - Duration of optimization doesn’t introduce delay in simulated closed loop.

5) Control Oriented Model of the Reaction Wheel Pendulum

- State Space Model [4]:
  \[
  \begin{align*}
  \dot{x}_1 &= x_2 \\
  \dot{x}_2 &= \frac{1}{K_a}(m g \sin(x_1) + K_s(u - k_{x0} - k_{x2})) \\
  \end{align*}
  \]
  - Extension for NMPC: \( u_i = \frac{1}{T}(u_{mpc} - u_i) \rightarrow x_{mpc} = [\dot{\theta} \dot{\phi} u_i]^T \)

6) Realization of the "Model Control Loop" control structure

- A NMPC (\( T_{sim} = 70ms \)) regulates the states of a sim. model online.
- Prediction of last step is state feedback at actual step (\( x_{mpc} \rightarrow x_{mpc}^{\text{ref}} \)).
- The interpolation between \( x_{mpc}^{\text{ref}} \) and \( x_{mpc}^{\text{ref}} \) is provided as a reference to the flatness based trajectory tracking controller (\( T_{last} = 4ms \)).

7) The NMPC-"Model Control Loop"

- Optimal Control Problem (OCP):
  \[
  \begin{align*}
  \min_{u_{mpc}} \int_0^T \|x_{mpc}(\tau) - x_{ref}(\tau)\|^2 + ||u_{mpc}(\tau) - u_{ref}(\tau)||^2 \ d\tau \\
  \text{s.t.} \quad y_{mpc}(\tau) = f_{mpc}(x_{mpc}(\tau), u_{mpc}(\tau)), \\
  \end{align*}
  \]
- Solution of the OCP with an Active-Set strategy and Newton-Iteration.
- Derived with ACADOtoolkit, using suboptimal realtime iteration [5].

8) Flatness Based Trajectory Tracking

- A flat output of the reaction wheel pendulum is \( y = \frac{1}{m} \dot{x}_2 + x_2 \).
- The Lie-derivatives along \( f(x, u) \) up to \( n = 3 \) are
  \[
  \begin{align*}
  \dot{y} &= L_{\lambda}(x) \frac{mg \sin(x_1)}{2} \\
  \ddot{y} &= L_{\lambda}(x) \frac{mg \cos(x_1)}{2} \\
  \end{align*}
  \]
- These equations define the transformation \( [x, u] \leftrightarrow [y, \dot{y}, \ddot{y}] \).
- Solving for \( u \) gives the feedback linearization \( u = \Psi(y, \dot{y}, \ddot{y}, \nu) \).
- The feedback control is \( \nu = \ddot{y} - k_{x2}(\dot{x}_2 - \dot{x}_{ref}) - k_{x1}(y - y_{ref}) \).

9) Experimental Result - Swingup of Reaction Wheel Pendulum

- Optimal reference trajectories by NMPC-"Model Control Loop".
- A flatness based controller is tracking the references.
- State estimation is based on a high gain observer.

10) References