Numerical approaches in a problem of management of hydroelectric resources

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Introduction

This work focuses on the study of a system of hydraulic power stations for which the energy production must be optimized.

The fluxes of water to turbine or pump on each power station are the control variables and the maximization of the profit of energy sale is the objective function.

Problem (P)

\[
\begin{align*}
\text{maximize} \quad & \int_0^T \left[ u(t) \left( \frac{V_i(t)}{N_i + 1} + H_i - \frac{V_i(t)}{N_i + 1} - \frac{H_i}{N_i} \right) + \nu(t) \left( \frac{V_i(t)}{N_i + 1} - H_i \right) \right] \, dt \\
\text{subject to} \quad & V_i(t) = A - u_i(t), \quad V_{i+1}(t) = V_i(t) + \nu(t), \quad i = 0, 1, \ldots, N-1, \\
& V(0) = V_0, \quad V(T) = V_f, \quad T = 1, 2, \ldots, N-2, \\
& u_i(t) \in [a_i, b_i], \quad i = 0, 1, 2, \ldots, N-1.
\end{align*}
\]

Objective: find a global solution to (DP)

Figure 1: Cascade of reversible hydro-electric power stations

1st Numerical Approach

Directly apply Chen-Burer algorithm to (DP)

2nd Numerical Approach

Define \( z = (b, y) \). Let \( \bar{x} = (x, z) \), \( \bar{a} = (a, 1) \).

Projected Problem (PP)

\[
\begin{align*}
(\bar{a}, \bar{x}) \rightarrow (x, \bar{Q} \bar{x}) \rightarrow \min, \\
x \in \bar{\Pi}
\end{align*}
\]

\( \bar{\Pi} \): approximated projection of feasible set for (DP) on the subspace of variables \((x, z) = (V_i(0), V_i(N-2), V_i(N-2), V_i(N-2), V_i(N-2), V_i(N-2)) \)

Technique: PER method (exterior approximation)

Chen-Burer algorithm is applied to (PP) \( \rightarrow (\bar{x}, \bar{z}) \)

Approximated solution to (DP) is obtained from:

\[
\begin{align*}
\text{minimize} \quad & \left\| \bar{\Pi}(y) - \bar{x} \right\|^2, \\
& A_y \leq b, \\
& A_{\bar{z}} y = b_{\bar{z}}, \\
& L_{\bar{y}} \leq y \leq U_{\bar{y}}.
\end{align*}
\]

Technique: QuadProg

Solution used as an initial guess for the local optimization package from [1]

Discretized Problem (DP)

\[
\begin{align*}
\text{minimize} \quad & j(x, y) = (a, x) + (b, y) + (x, Qx) \\
V_i(k) \in [a_i, b_i], \quad & \text{for } k = 0, \ldots, N-1 \text{ and } i = 1, 2, \\
V_i(k) + A - V_i(k+1) \in [a_i, b_i], \quad & \text{for } k = 0, \ldots, N-2, \\
V_i(k) + A - V_i(k+1) - V_i(k+1) \in [a_i, b_i], \quad & \text{for } k = 0, \ldots, N-2, \\
V_i(N-1) + A - V_i(0) \in [a_i, b_i], \\
V_i(0) + A - V_i(0) \in [a_i, b_i].
\end{align*}
\]

Case Study

Data:

\[
\begin{align*}
V_0 &= 86.7, \quad V_f = 147, \quad V_M = 48.3, \quad V_M = 66, \\
& u_1 = 0, \quad u_2 = 0.8316, \quad u_3 = -0.3456, \quad u_4 = 0.4392, \\
& N = 24, \quad c_1 = 2, \quad c_2 = 20, \quad H_1 = 3, \quad H_2 = 1, \\
& A = 0.1589, \quad S_1 = 81.7, \quad S_2 = 44.5.
\end{align*}
\]

Conclusion

The configuration of the cascade can be arbitrarily complex. Here we considered an illustrative example with only two stations.

Remark

\[ \text{Construction of the convex hull of a finite set of points when the computations are approximate.} \]

\[
\begin{align*}
\text{External Estimation Construction (described by support planes):}
\end{align*}
\]

References


[3] O.L. Chernykh. \textit{Construction of the convex hull of a finite set of points when the computations are approximate}. 


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