Preditive Control in an Irrigation Planning Problem

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Abstract

We address the problem of optimizing the water use in the irrigation of farm fields by means of optimal control. To correct any discrepancies and to get the best estimate of the volume of irrigation water consumed with current weather data we have used preditive control techniques. In particular, we replan our problem using the latest precipitation data available.

The Problem

$$
\begin{aligned}
&\min \sum_{i=1}^{N-1} u_i \\
&\text{s.t.: } x_{i+1} = x_i + h f(t_i, x_i, u_i) \quad \text{a.e. } i = 1, \ \ldots, \ N-1 \\
&\qquad x_i \geq x_{\min} \quad i = 1, \ \ldots, \ N \\
&\qquad u_i \geq 0 \quad \text{a.e. } i = 1, \ \ldots, \ N-1 \\
&\qquad x_1 = x_0,
\end{aligned}
$$

f is the balance hydrologic function, $x_{\min}$ is the hydrological need of the crop, x_0 is an initial state, h is the time step discretization, and $N = 12/h$.

$$f(t_i, x_i, u_i) = u_i + \text{rainfall}(t_i) - \text{evapotranspiration}(t_i) - \text{losses}(x_i).$$

The state constraint ($x_i \geq x_{\min}$) is based on the fact that the plant needs a minimal amount of water to survive.

Replan

Due to the unpredictability of weather conditions, the general model presented in the previous section may not describe accurately the system. To overcome this drawback, we developed a model based on replan. We test the replan model for the last ten years and we observed that state constraint was violated in the years 2003, 2005, 2007, 2008, and 2009. For instance considering the year **2008**.

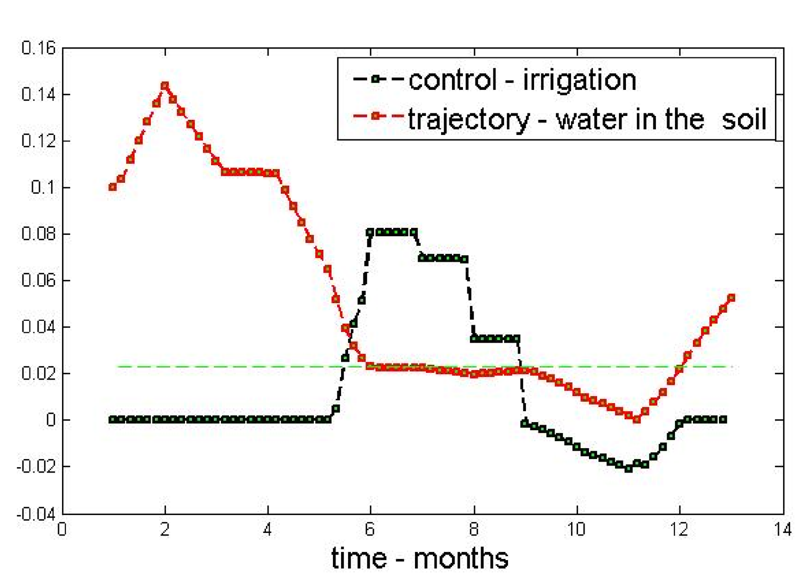


Figure 1: Results for the replan planning for the year 2008.

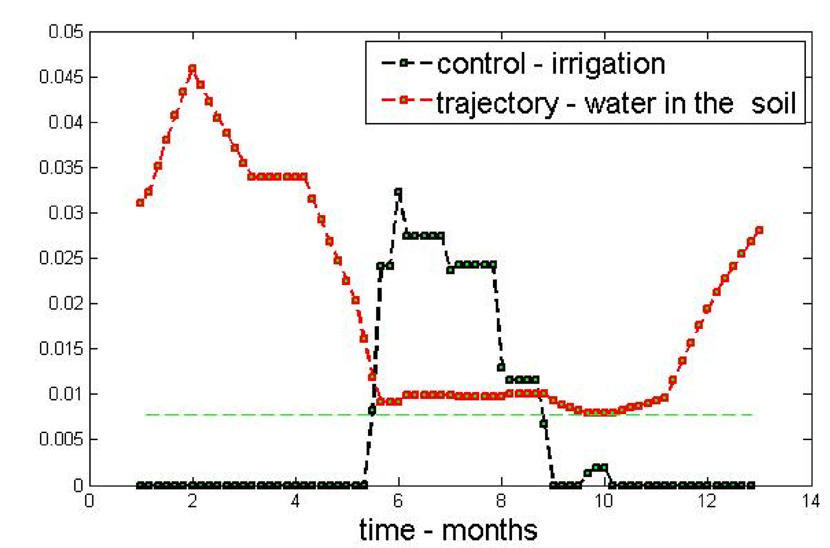
New Model

To overcome this obstacle a new model had to be considered. In this new model, we use soft state constraints instead of hard state constraint. Therefore, we consider an optimal control problem with a penalization in the cost function. This cost penalization is zero if the state constraint is not violated and increases exponential otherwise, where this state constraint has a safety margin. We impose minimum amount of water a bit higher so that the true constraint is not violated because of weather conditions.

Results

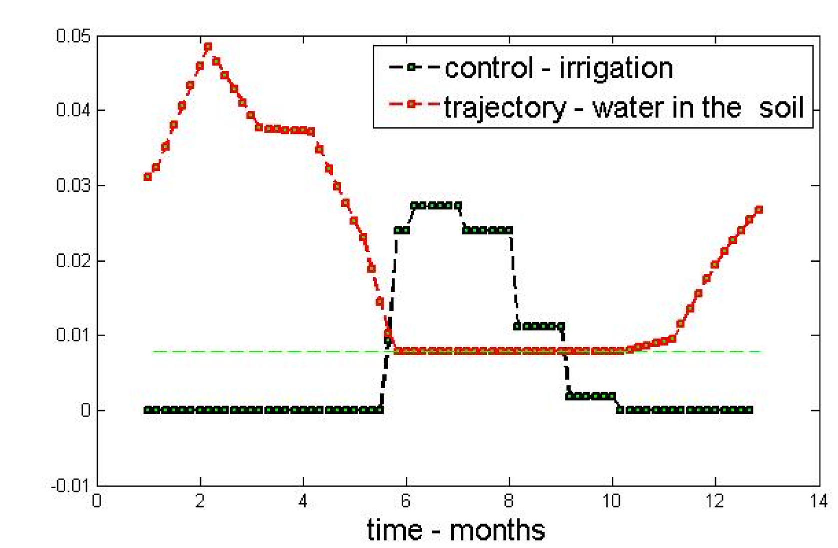
The next figures compare the results for a field of potatoes in the region of Lisbon with unit area considering that we know a priori the real data of rainfall of 2008 with the result obtained using the new model.

Replan in 2008



Optimal Solution is 0.4418 m^3/year.

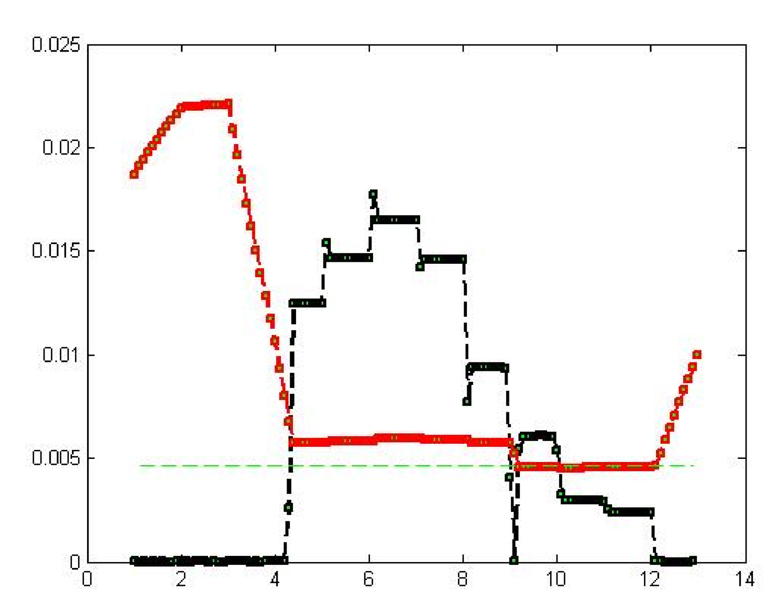
Rainfall Known a Priori in 2008



Optimal Solution is 0.4416 m^3/year.

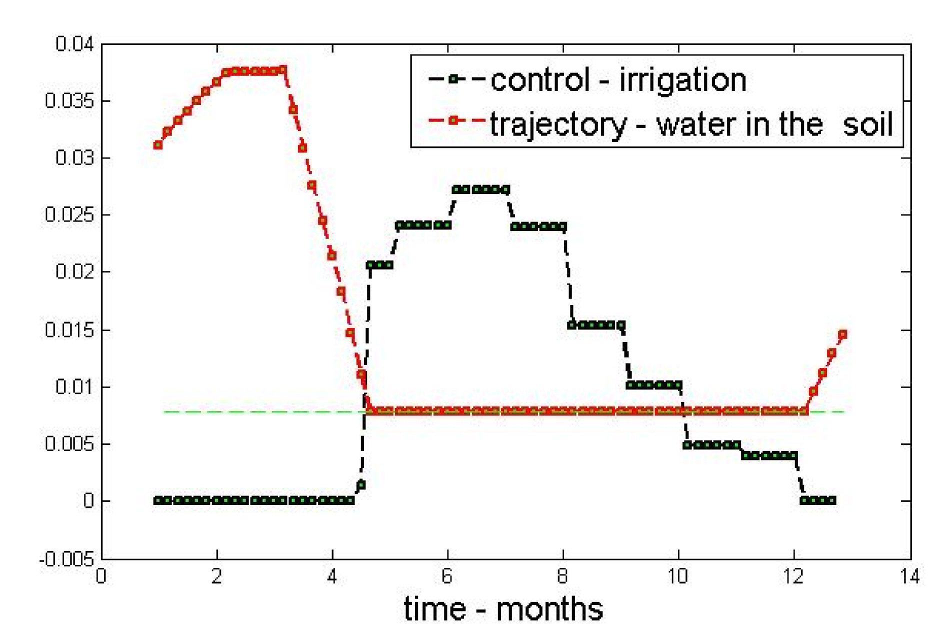
The next figures compare the results for a field of potatoes in the region of Lisbon with unit area considering that we know a priori an atypically hypothetical year of rainfall (do not rain between april and december) with the result obtained using the new model.

Replan



Optimal Solution is 0.7425 m^3/year.

Rainfall Known a Priori



Optimal Solution is 0.7210 m^3/year.

Conclusions

The predictive control techniques combined with a penalization of cost function allows us to overcome the obstacles that have arisen with the planning of the irrigation problem under real data.

References

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[2] F. A. C. C. Fontes, L. Magni. *Min-Max Model Predictive Control of Nonlinear Systems Using Discontinuous Feedbacks.* IEEE Transactions on Automatic Control, vol. 48, no. 10, pp. 1750 - 1755, 2003.

[3] J. R. Raposo, *A REGA - dos primitivos regadios nas modernas tecnicas de rega.* Fundação Calouste Gulbenkian, 1996.

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