OPTEC
K.U.Leuven Optimization in Engineering Center

Robust Model Predictive Control Formulation for Systems with Polytopic Uncertainty

Adeleh Mohammadi, Joris Gillis, Mario Zanon, and Moritz Diehl

ESAT/SCD

Abstract

In this poster, we consider a min-max model predictive Control (MPC) problem with convex cost and constraints for a linear system with polytopic uncertainty. The controller is designed to be able to control all the possible systems with parameters that can vary inside a polytope. The problem is formulated as Quadratically Constrained Quadratic Program (QCQP). Since this approach is based on a scenario tree formulation, the number of variables grows exponentially with the horizon length. The QCQP is then solved using an interior-point method. The simulation result is then compared to a nominal MPC formulation. It is observed that the nominal MPC results in infeasibility of the optimization problem while the robust MPC controller can deal with uncertainties and the system has a stable closed-loop response.

Nominal MPC

Formulation of Nominal MPC with terminal cost and constraints:

$$\underset{x_0,\ldots x_N,u_0,\ldots u_{N-1}}{\text{minimize}} \quad \sum_{k=0}^{N-1} x_k^T Q x_k + u_k^T R u_k + x_N^T Q_N x_N \qquad (1a)$$

$$\text{subject to} \quad x_0 = \hat{x}_0 \qquad (1b)$$

$$x_{k+1} = A x_k + B u_k \quad \forall k = 0,\ldots,N-1 \qquad (1c)$$

$$F x_k + G u_k \le g \quad \forall k = 0,\ldots,N-1 \qquad (1d)$$

$$x_k^T P_N x_k \le p_N \quad k = N \qquad (1e)$$

Robust MPC with Quadratic Costs

Polytopic system: Linear and of the form

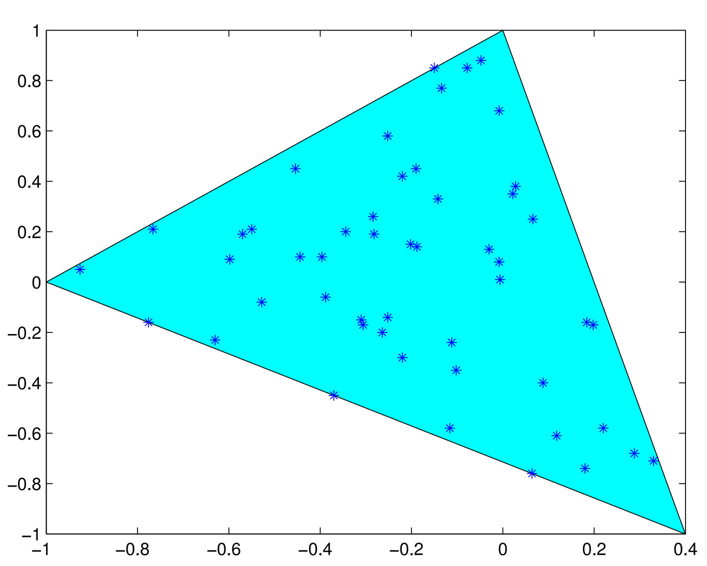
$$f(x,u) = Ax + Bu + c \qquad (2)$$

The matrices A and B and the vector c are contained in a polytope with n_f vertices, F is the convex hull of $f_1,\ldots,f_{n_f}$.

$$F = \text{conv}\{(A_1|B_1|c_1),\ldots,(A_{n_f}|B_{n_f}|c_{n_f})\} \qquad (3)$$

Polytopic Uncertainty Realization:

- Each vertex represents one system which results in the polytopic uncertainty region.
- At each step, a uniformly random convex combination of vertices inside a polytope is chosen.



Polytopic uncertainty.

$$x_{k+1} = A x_k + B u_k + c \qquad (4)$$

$$A = \textstyle\sum_{i=1}^{m} \alpha_i A_i,$$
$$B = \textstyle\sum_{i=1}^{m} \alpha_i B_i,$$
$$c = \textstyle\sum_{i=1}^{m} \alpha_i c_i,$$
$$\textstyle\sum_{i=1}^{m} \alpha_i = 1, \alpha_i \ge 0, \quad i = 1,\ldots m.$$

The problem is to find a set of controls for the system with parameters varing inside the polytope.

Min-max MPC as QCQP:

$$\underset{u_j,v_j(j\in\mathcal{P}),x_j(j\in\mathcal{C})}{\text{minimize}} \quad x_0^T Q x_0 + u_0^T R u_0 + v_0 \qquad (5a)$$

$$\text{subject to} \quad F x_j + G u_j \le g \quad \forall j \in \mathcal{P} \qquad (5b)$$

$$A_{\nu(j)} x_{p(j)} + B_{\nu(j)} u_{p(j)} + c_{\nu(j)} = x_j \quad \forall j \in \mathcal{C} \qquad (5c)$$

$$x_j^T Q x_j + u_j^T R u_j + v_j \le v_{p(j)} \quad \forall j \in \mathcal{C} \cap \mathcal{P} \qquad (5d)$$

$$x_j^T Q_N x_j - v_{p(j)} \le 0 \quad \forall j \in \mathcal{C} \setminus \mathcal{P} \qquad (5e)$$

$$x_j^T P_N x_j \le p_N \quad \forall j \in \mathcal{C} \setminus \mathcal{P} \qquad (5f)$$

The uncertainty is propagated using a scenario tree.

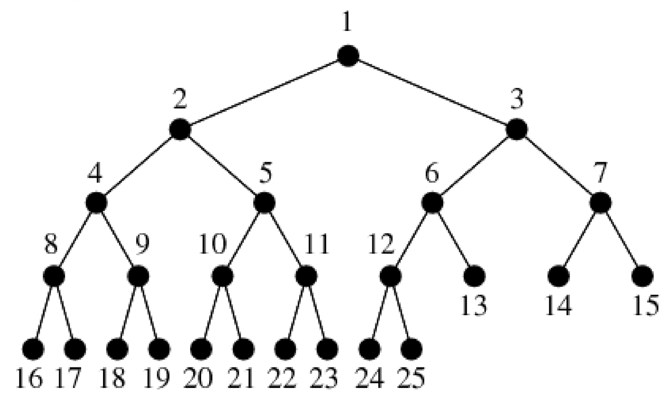


Figure 1: *Scenario tree.*

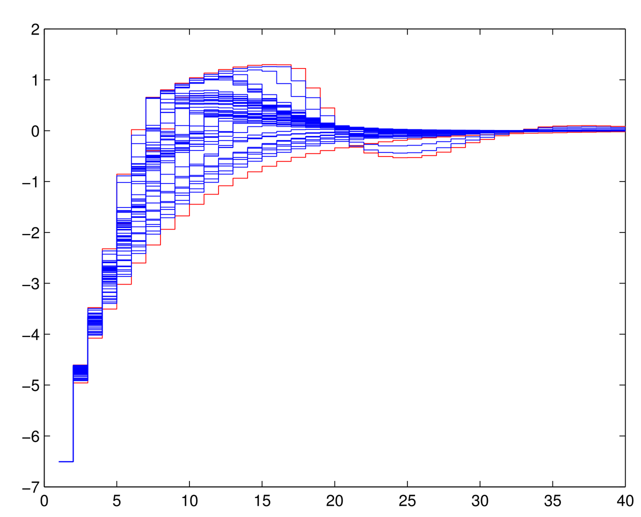
Simulation

Nominal MPC:

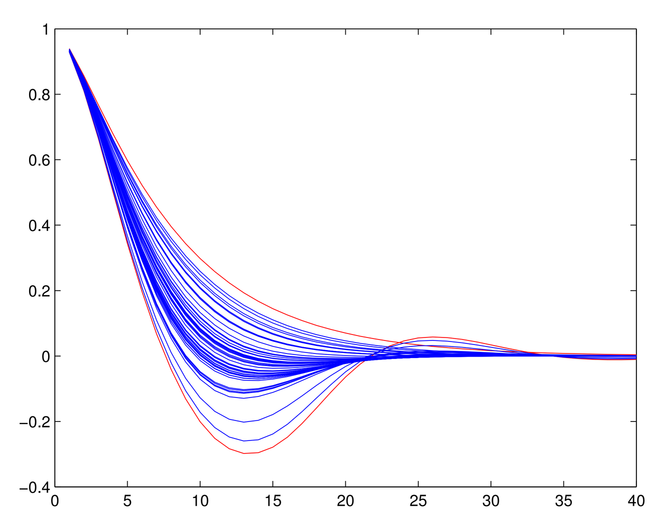
- Solving the nominal MPC and applying the controller to some random realization inside the triangle.
- The nominal MPC problem runs into *infeasibility* and cannot deal with the uncertainties.

Robust MPC:

- Designing the controller based on three extreme cases.
- Uncertainty is propagated through the MPC horizon.
- The number of stages (MPC horizon) is $m = 5$.
- Total number of nodes: $N_C = \sum_{i=0}^{m} n_f^i = 364$.
- The optimization problem is a QCQP solved using $cplex$.
- Closed-loop simulation for 50 different uncertainty realizations.



(a) Input.



(b) State trajectory.