Numerical approaches in a problem of management of hydroelectric resources

A. F. Ribeiro*, V. A. Bushenkov†, M. M. A. Ferreira* and G. V. Smirnov§

*University of Porto - FEUP, †University of Évora, §University of Minho

Introduction

This work focuses on the study of a system of hydroelectric power stations for which the energy production must be optimized.

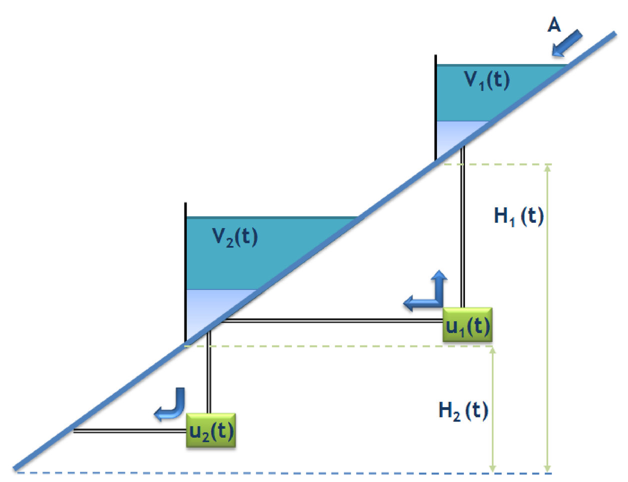


Figure 1: Cascade of reversible hydro-electric power stations

The fluxes of water to turbine or pump on each power station are the control variables and the maximization of the profit of energy sale is the objective function.

Problem (P)

maximize

$$\int_0^T c(t)\left[u_1(t)\left(\frac{V_1(t)}{S_1}+H_1-\frac{V_2(t)}{S_2}-H_2\right)+u_2(t)\left(\frac{V_2(t)}{S_2}-H_2\right)\right]dt$$

$$\dot{V}_1(t)=A-u_1(t),\quad \dot{V}_2(t)=u_1(t)-u_2(t)$$

$$V_i(0)=V_i(T),\ \text{for } i=1,2$$

$$V_i(t)\in\left[V_i^m,V_i^M\right],\ \text{for } i=1,2$$

$$u_i(t)\in\left[u_i^m,u_i^M\right],\ \text{for } i=1,2$$

Discretized Problem (DP)

minimize

$$I(x,y)=\langle a,x\rangle+\langle b,y\rangle+\langle x,Qx\rangle$$

$$V_i(k)\in\left[V_i^m,V_i^M\right],\ \text{for } k=0,\cdots,N-1 \text{ and } i=1,2,$$

$$V_1(k)+A-V_1(k+1)\in\left[u_1^m,u_1^M\right],\ \text{for } k=0,\cdots,N-2,$$

$$V_2(k)+V_1(k)+A-V_1(k+1)-V_2(k+1)\in\left[u_2^m,u_2^M\right],$$

$$\text{for } k=0,\cdots,N-2,$$

$$V_1(N-1)+A-V_1(0)\in\left[u_1^m,u_1^M\right],$$

$$V_2(N-1)+V_1(N-1)+A-V_1(0)-V_2(0)\in\left[u_2^m,u_2^M\right].$$

where $x=(V_1(0),V_1(N/2),V_2(0),V_2(N/2))$,
$y=(V_1(1),\cdots,V_1(N/2-1),V_1(N/2+1),\cdots,V_1(N-1),$
$V_2(1),\cdots,V_2(N/2-1),V_2(N/2+1),\cdots,V_2(N-1))$
and Q is an indefinite matrix.

Objective: find a global solution to (DP)

1st Numerical Approach

Directly apply Chen-Burer algorithm to (DP)

2nd Numerical Approach

Define $z=\langle b,y\rangle$. Let $\bar{x}=(x,z)$, $\bar{a}=(a,1)$.

Projected Problem (PP)

$$\langle\bar{a},\bar{x}\rangle+\langle\bar{x},\bar{Q}\bar{x}\rangle\to\min,$$
$$\bar{x}\in\overline{\Pi}$$

1. $\overline{\Pi}$: approximated projection of feasible set for (DP) on the subspace of variables
$$(x,z)=(V_1(0),V_1(N/2),V_2(0),V_2(N/2),z).$$
Technique: PER method(exterior approximation)
2. Chen-Burer algorithm is applied to (PP) $\to(\hat{x},\hat{z})$
3. Approximated solution to (DP) is obtained from:
$$\begin{aligned}\text{minimize}\quad & \|\Pi(y)-\hat{x}\|^2,\\ & Ay\le b,\\ & A_{eq}y=b_{eq},\\ & LB\le y\le UB,\end{aligned}$$
Technique: *QuadProg*
4. Solution used as an initial guess for the local optimization package from [1]

Case Study

Data:

$$V_1^m=86.7,\ V_1^M=147,\ V_2^m=48.3,\ V_2^M=66,$$
$$u_2^m=0,\ u_2^M=0.8316,\ u_1^m=-0.3456,\ u_1^M=0.4392,$$
$$N=T=24,\ c_1=2,\ c_2=20,\ H_1=3,\ H_2=1,$$
$$A=0.1589,\ S_1=81.7,\ S_2=44.5.$$

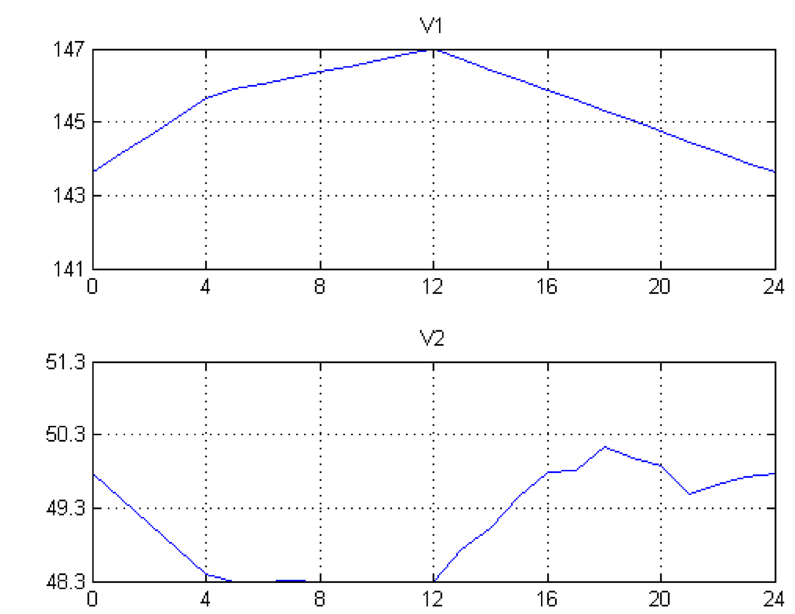
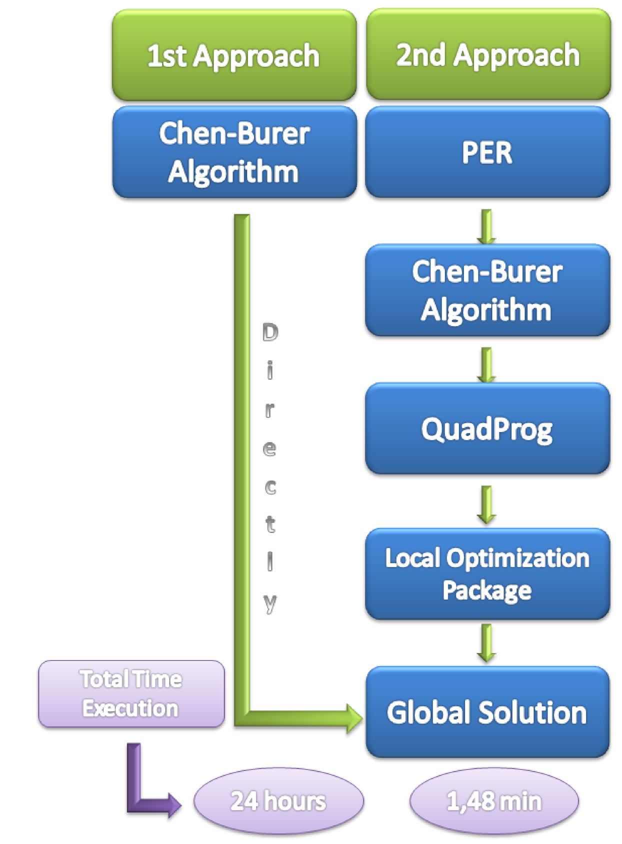


Figure 2: Global Solution

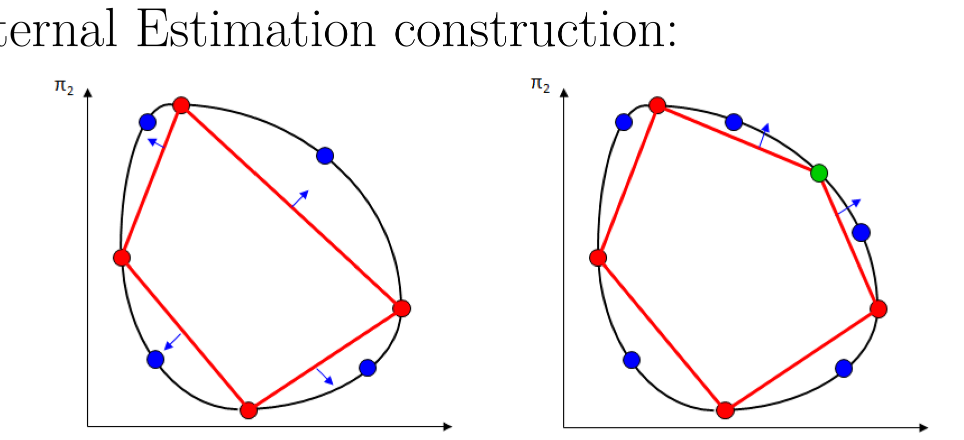
Conclusion



- Solutions: V_2 may differ on the second half part of the interval. Non-unique optimal solution
- Cost: the same for the two approaches
- Time taken by the 1st approach is very long when compared with the second

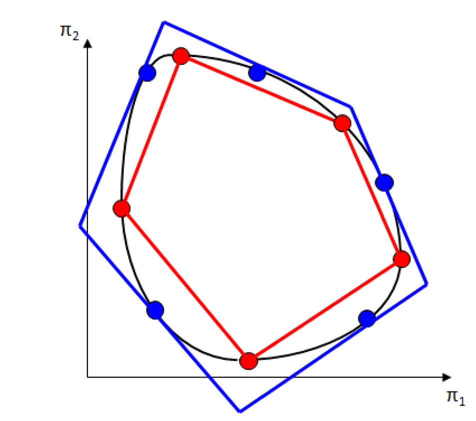
Projection Estimation Refined (PER) Method

- Internal Estimation construction:



(a) 1st iteration - the initial set (b) 2nd iteration - the most distant new point was included into the convex hull

- External Estimation Construction (described by support planes):



Remark

The configuration of the cascade can be arbitrarily complex. Here we considered an illustrative example with only two stations.

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