ESTIMATES ON THE PREDICTION HORIZON LENGTH IN MODEL PREDICTIVE CONTROL*

K. WORTHMANN†

Abstract. We are concerned with model predictive control without stabilizing terminal constraints or costs. Here, our goal is to determine a prediction horizon length for which stability or a desired degree of suboptimality is guaranteed. To be more precise, we extend the methodology introduced in [7] in order to improve the resulting performance bounds. Furthermore, we carry out a comparison with other techniques designed for deriving estimates on the required prediction horizon length.

Key words. Stabilization, model predictive control without terminal constraints, performance estimates, required prediction horizon length, optimization based control, nonlinear control systems.

AMS subject classifications. 34H15 93D15 93C10 93C57 93B51

1. Introduction.

Model predictive control (MPC), also termed receding horizon control (RHC), is a well established control strategy in order to approximately solve optimal control problems on an infinite time horizon, e.g. stabilization of a given control system at a desired set point, cf. [3, 1, 22]. To this end, a sequence of optimal control problems on a truncated and, thus, finite time horizon is generated in order to compute a feedback. Since this methodology allows to directly incorporate constraints and is applicable to nonlinear as well as infinite dimensional systems, cf. [16, 8, 12], MPC has attracted considerable attention during the last decades, cf. [10]. However, the stability analysis of MPC is far from being trivial, cf. [21]. Often, additional (artificial) terminal constraints or costs are used in order to ensure stability and feasibility of the MPC closed loop, cf. [14, 4]. However, since these stability enforcing modifications of the underlying optimization problems are seldomly used in practice, we concentrate on so called unconstrained MPC schemes, cf. [20]. Here, unconstrained indicates that neither additional terminal constraints nor costs are incorporated in the formulation of the auxiliary problems of the corresponding MPC schemes.

Stability and feasibility were shown for unconstrained MPC for “sufficiently large” prediction horizon, cf. [13]. Since the length of the prediction horizon predominantly determines the numerical effort needed in order to solve the optimal control problem in each MPC iteration, this result automatically leads to the question of how to determine this quantity suitably. A technique in order to deal with this issue can be found in [17]. However, the proposed methodology is only applicable for linear, finite dimensional systems without control or state constraints. For nonlinear constrained systems a first approach is given in [6] which was significantly improved in [23]. A more recent approach, which is also applicable for infinite dimensional systems, was introduced in [7] and further elaborated in [9]. Both approaches have in common that a controllability assumption is exploited in order to estimate a prediction horizon length for which asymptotic stability or even a desired performance in comparison to the optimal solution on the infinite time horizon is guaranteed. However, the assumed conditions deviate. Here, we extend the methodology from [7, 9] to the weaker assumption from [23] which allows to ensure the same performance bounds for significantly shorter prediction horizons in comparison to [23]. Furthermore, we

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†University of Bayreuth, 95440 Bayreuth, Germany, karl.worthmann@uni-bayreuth.de.

illustrate how this generalization can be employed in order to further tighten the estimates on the horizon length from [7].

The paper is organized as follows. In Section 2 MPC is introduced and the methodology proposed in [7, 9] is concisely summarized. In the ensuing section this technique is adapted to the weaker controllability assumption from [23]. Based on this result a comparison to the prior approaches stemming back to [23, 7] is drawn. In order to illustrate our results, the example of the synchronous generator taken from [5] is considered. Some conclusions are given in Section 5.

2. Model Predictive Control. Let $\mathbb{N}$ and $\mathbb{R}$ denote the natural and the real numbers, respectively. Additionally, the definition $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$ is used and a continuous function $\eta : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ which is strictly monotone, unbounded, and satisfies $\eta(0) = 0$ is said to be of class $\mathcal{K}_\infty$.

Nonlinear discrete time control systems governed by system dynamics

$$x(n+1) = f(x(n), u(n)), \qquad x(0) = x_0, \tag{2.1}$$

with state space X and set U of control values are considered. Here, X and U are normed Banach spaces. For a given sequence of control values $u = u(n)_{n\in\mathbb{N}_0}$, $x_u(n; x_0) = x(n; x_0, (u(n))_{n\in\mathbb{N}_0})$ denotes the trajectory emanating from the initial state x_0 and evolving according to (2.1). State and control constraints are modeled by suitably chosen subsets $\mathbb{X} \subseteq X$ and $\mathbb{U} \subseteq U$, respectively. A sequence of control values $u = (u(n))$, $n = 0, 1, 2, \ldots, N-1$, is called admissible for $x_0 \in \mathbb{X}$, i.e. $u \in \mathcal{U}^N(x_0)$, if the conditions

$$x_u(n+1; x_0) \in \mathbb{X} \qquad \text{and} \qquad u(n) \in \mathbb{U}$$

hold for each $n \in \{0, 1, 2, \ldots, N-1\}$. Furthermore, $u = (u(n))_{n\in\mathbb{N}_0}$ is said to be admissible if, for each $N \in \mathbb{N}$, the restriction to its first N elements is contained in $\mathcal{U}^N(x_0)$. Then, we write $u \in \mathcal{U}^\infty(x_0)$. We assume that the set $\mathbb{X}$ is control invariant, i.e. for each $x \in \mathbb{X}$ a control value $u \in \mathbb{U}$ exists such that $f(x, u) \in \mathbb{X}$ holds, cf. [19, 15] and [8, Sections 8.2 and 8.3]. This is, in turn, equivalent to assuming $\mathcal{U}^1(x) \neq \emptyset$ for all $x \in \mathbb{X}$. Iterative application of this condition allows to infer $\mathcal{U}^\infty(x) \neq \emptyset$.

Let a desired set point $x^\star \in \mathbb{X}$ and a control value $u^\star \in \mathbb{U}$ satisfying $f(x^\star, u^\star) = x^\star$ be given. Furthermore, let running costs $\ell : X \times U \to \mathbb{R}_0^+$ be defined such that

$$\ell(x^\star, u^\star) = 0 \qquad \text{and} \qquad \ell(x, u) \geq \eta(\|x - x^\star\|) \quad \forall\, x \in \mathbb{X} \tag{2.2}$$

hold for some $\eta \in \mathcal{K}_\infty$. Our goal is to minimize the cost functional $J_\infty(x_0, u) := \sum_{n=0}^{\infty} \ell(x_u(n; x_0), u(n))$ with respect to $u = (u(n))_{n\in\mathbb{N}_0} \in \mathcal{U}^\infty(x_0)$. Since optimal control problems on an infinite time horizon are, in general, computationally intractable, MPC is employed in order to approximate the solution or, at least, to stabilize the considered system at $x^\star$. To this end, the optimization problem is solved on a truncated and, thus, finite horizon:

$$\text{Minimize} \quad J_N(\bar{x}, u) := \sum_{n=0}^{N-1} \ell(x_u(n; \bar{x}), u(n)) \tag{2.3}$$

with respect to $u \in \mathcal{U}^N(\bar{x})$ and $\bar{x} = x_0$. The corresponding optimal value function $V_N(\cdot)$ is given by $V_N(\bar{x}) := \inf_{u\in\mathcal{U}^N(\bar{x})} J_N(\bar{x}, u)$. In order to keep the presentation technically simple, this infimum is assumed to be a minimum. Computing an optimal sequence of control values

$$u^\star_{\bar{x}} = (u^\star_{\bar{x}}(0), u^\star_{\bar{x}}(1), u^\star_{\bar{x}}(2), \ldots, u^\star_{\bar{x}}(N-1)) \in \mathcal{U}^N(\bar{x})$$

satisfying $J_N(\bar{x}, u^\star_{\bar{x}}) = V_N(\bar{x})$ allows us to define a feedback map $\mu_N : X \to U$ by setting $\mu_N(\bar{x}) := u^\star_{\bar{x}}(0) \in \mathbb{U}$ which yields the successor state

$$x_1 := f(x_0, \mu_N(x_0)) = f(\bar{x}, \mu_N(\bar{x})) = f(\bar{x}, u^\star_{\bar{x}}(0)) \in \mathbb{X}.$$

Then, the optimization horizon is shifted forward in time and the optimization Problem (2.3) is solved for the new initial state $\bar{x} = x_1$. Iterative application of this procedure generates a closed loop control on the infinite time horizon. The corresponding trajectory is denoted by $x_{\mu_N}(n; x_0)$, $n \in \mathbb{N}_0$.

Stability of such MPC schemes can be ensured by a sufficiently large prediction horizon N, cf. [13]. In order to estimate the required horizon length, we suppose that the following controllability condition introduced in [7] holds.

Assumption 2.1 (Grüne). *Let a sequence $(c_n)_{n\in\mathbb{N}_0} \subset \mathbb{R}^+_0$ satisfying the submultiplicativity condition $c_n c_m \geq c_{n+m}$ for $n, m \in \mathbb{N}_0$ and $\sum_{n=0}^\infty c_n < \infty$ be given such that, for each $\bar{x} \in \mathbb{X}$, a sequence $u_{\bar{x}} \in \mathcal{U}^\infty(\bar{x})$ of control values exists which satisfies*

$$\ell(x_{u_{\bar{x}}}(n; \bar{x}), u_{\bar{x}}(n)) \leq c_n \min_{u \in \mathcal{U}^1(\bar{x})} \ell(\bar{x}, u) =: c_n \ell^\star(\bar{x}). \tag{2.4}$$

For instance, such a sequence may be defined by $c_n := C\sigma^n$ with overshoot $C \geq 1$ and decay rate $\sigma \in (0,1)$ for systems which are exponentially controllable in terms of their stage costs, cf. [2] for an example. Based on Assumption 2.1 the following Theorem can be deduced.

Theorem 2.2. *Let Assumption 2.1 be satisfied. Then, for each $\overline{\alpha} \in [0,1)$, a prediction horizon N can be chosen such that the condition*

$$\alpha_N := 1 - \frac{(\gamma_N - 1)\prod_{i=2}^N (\gamma_i - 1)}{\prod_{i=2}^N \gamma_i - \prod_{i=2}^N (\gamma_i - 1)} > \overline{\alpha} \qquad \text{with} \qquad \gamma_i := \sum_{n=0}^{i-1} c_n \tag{2.5}$$

is satisfied. Furthermore, for all $x \in \mathbb{X}$, the relaxed Lyapunov inequality

$$V_N(f(x, \mu_N(x))) \leq V_N(x) - \alpha_N \ell(x, \mu_N(x)) \leq V_N(x) - \overline{\alpha}\ell(x, \mu_N(x)) \tag{2.6}$$

holds for the MPC feedback μ_N with prediction horizon N. If, in addition to (2.2), $\varrho \in \mathcal{K}_\infty$ exists such that $V_N(x) \leq \varrho(\|x - x^\star\|)$ is satisfied on $\mathbb{X}$, asymptotic stability of the MPC closed loop and the following performance bound is guaranteed

$$J^{\mu_N}_\infty(x_0) := \sum_{n=0}^\infty \ell(x_{\mu_N}(n; x_0), \mu_N(x_{\mu_N}(n; x_0))) \leq \alpha_N^{-1} V_\infty(x_0). \tag{2.7}$$

Proof. [9, Corollary 6.1] ensures the existence of $N \in \mathbb{N}_{\geq 2}$ such that Condition (2.5) holds. Then, [9, Theorem 5.4] yields Inequality (2.6). As a consequence, [7, Theorem 5.2 and Proposition 2.4] can be applied in order to conclude asymptotic stability of the MPC closed loop with prediction horizon N . □

Summarizing Theorem 2.2 allows to easily compute a prediction horizon N for which stability or a certain degree of suboptimality of the introduced MPC scheme is guaranteed supposing Assumption 2.1. Note that Theorem 2.2 only requires $u_{x_0} \in \mathcal{U}^N(x_0)$ satisfying Inequality (2.4) for $n = 0, 1, 2, \ldots, N-1$, cf. [11].

3. Main Results. In this section we show that Theorem 2.2 and, thus, the methodology proposed in [7, 9] can also be applied supposing the following controllability assumption introduced in [23]. Then, the resulting suboptimality estimate is compared with its counterpart from [23].

ASSUMPTION 3.1 (Tuna, Messina, and Teel). *Let a monotone sequence $(M_i)_{i\in\mathbb{N}} \subset [1, M]$, $M < \infty$, be given such that, for each $x \in \mathbb{X}$, the following inequality holds*

$$V_i(x) \leq M_i \min_{u\in\mathcal{U}^1(x)} \ell(x, u) =: M_i\ell^\star(x) = M_iV_1(x). \tag{3.1}$$

Note that Assumption 3.1 is weaker than Assumption 2.1. In order to substantiate this claim, suppose that Inequality (2.4) holds which immediately implies

$$V_i(\bar{x}) \leq \sum_{n=0}^{i-1} \ell(x_{u_{\bar{x}}}(n; \bar{x}), u_{\bar{x}}(n)) \leq \sum_{n=0}^{i-1} c_n\ell^\star(\bar{x}) = \gamma_i\ell^\star(\bar{x})$$

and, thus, Condition (3.1) with $M_i = \gamma_i$. The other way round, such a conclusion is, in general, not possible. We want to replace the definition of γ_i, $i = 2, 3, 4, \ldots, N$, and, thus, α_N in Theorem 2.2 by $\gamma_i = M_i$ with M_i from Inequality (3.1).

THEOREM 3.2. *Suppose that Assumption 3.1 is satisfied. Then, the assertions of Theorem 2.2 still hold based on Formula (2.5) applied with $\gamma_i := M_i$, $i \in \mathbb{N}_{\geq 2}$.*

Proof. Without loss of generality $M_1 = 1$ is assumed. Otherwise the sequence $(M_i)_{i\in\mathbb{N}}$ from Inequality (3.1) may be suitably adapted. Then, a so called equivalent sequence $(c_n)_{n\in\mathbb{N}_0}$ can be defined by $c_0 = 1$ and $c_i := M_{i+1} - M_i$. If the submultiplicativity condition $c_nc_m \geq c_{n+m}$ holds for this equivalent sequence, the proof of Theorem 2.2 does not need to be changed because neither [9, Theorem 5.4] nor [9, Corollary 6.1] require the exact shape of the involved sequence $(c_n)_{n\in\mathbb{N}_0}$ but rather the accumulated bounds $(\gamma_i)_{i\in\mathbb{N}_{\geq 2}}$, which are given by the sequence $(M_i)_{i\in\mathbb{N}}$.

If the submultiplicativity condition is violated, Theorem 2.2 still provides a lower bound according to [9, Remark 5.5]. Indeed, the estimate may even be tightened by solving the corresponding linear program given in [7]. □

We point out that the proof of Theorem 3.2 is mainly based on an observation. However, the concluded assertion allows to significantly tighten our performance estimates as will be shown in the ensuing section. Note that the used concept of an equivalent sequence does, in general, not take account of Condition (2.4).

Next, we compare the presented technique with the methodology introduced in [23]. Note that this approach allows to incorporate a (control) Lyapunov function as a terminal weight in the MPC cost functional $J_N(\cdot)$. However, since constructing a suitable terminal cost is, in general, a challenging task for nonlinear, constrained systems, we do not want to make use of this option. Hence, the additional condition $\ell^\star(f(x, u)) + \ell(x, u) \leq (1 + \kappa)\ell^\star(x)$ is automatically satisfied with $\kappa := M_2 - 1$. Then, the suboptimality degree in the relaxed Lyapunov Inequality (2.6) is given by

$$\widetilde{\alpha}_N := 1 - \kappa \cdot (M_N - 1) \prod_{i=2}^{N-1} \frac{M_i - 1}{M_i}, \tag{3.2}$$

cf. [23, Theorem 1]. Proposition 3.3 shows that the obtained performance bounds from Theorem 3.2 are tighter than those resulting from (3.2). The reason is the deduction of the respective formulas. In order to derive (2.5) additional inequalities were taken into account, cf. [24, Section 5.5]. This indicates that the assertion of Proposition

3.3 also holds without the assumed submultiplicativity condition. However, then the linear program proposed in [7] has to be solved instead of using Formula (2.5).

PROPOSITION 3.3. *Let a monotone bounded sequence $(M_i)_{i\in\mathbb{N}_{\geq 2}} = (\gamma_i)_{i\in\mathbb{N}_{\geq 2}}$ be given such that the equivalent sequence $(c_n)_{n\in\mathbb{N}_0}$ constructed analogously to the proof of Theorem 3.2 satisfies the submultiplicativity condition $c_n c_m \geq c_{n+m}$, $n,m \in \mathbb{N}_0$. Then, the inequality $\alpha_N \geq \widetilde{\alpha}_N$ holds for α_N from Formula (2.5) and $\widetilde{\alpha}_N$ from Formula (3.2) with $\kappa = \gamma_2 - 1$ for all $N \in \mathbb{N}_{\geq 2}$.*

Proof. The assertion $\alpha_N \geq \widetilde{\alpha}_N$ is equivalent to

$$(\gamma_N - 1)\prod_{i=2}^{N-1}\gamma_i \leq (\gamma_2 - 1)\left[\prod_{i=2}^{N}\gamma_i - \prod_{i=2}^{N}(\gamma_i - 1)\right], \tag{3.3}$$

an inequality which we show by induction with respect to $N \in \mathbb{N}_{\geq 2}$. For $N = 2$, (3.3) holds with equality, i.e. the induction start. Next, we carry out the induction step from $N \rightsquigarrow N+1$. For $N+1$, the right hand side of (3.3) can be rewritten as

$$\gamma_{N+1}(\gamma_2 - 1)\left[\prod_{i=2}^{N}\gamma_i - \prod_{i=2}^{N}(\gamma_i - 1)\right] + (\gamma_2 - 1)\prod_{i=2}^{N}(\gamma_i - 1)$$

which allows to use the induction assumption. Hence, ensuring the inequality

$$\underbrace{(\gamma_2 - 1)}_{=c_1+c_0-1\geq c_1}(\gamma_2 - 1)(\gamma_N - 1)\prod_{i=3}^{N-1}(\gamma_i - 1) - \underbrace{(\gamma_{N+1} - \gamma_N)}_{=c_N\leq c_1 c_{N-1}}\prod_{i=2}^{N-1}\gamma_i \geq 0 \tag{3.4}$$

is sufficient in order to prove (3.3). Factoring c_1 out and applying [9, Lemma 10.1] with $j = k = m = \omega = 1$ yields (3.4) and, thus, completes the proof. ◻

In both settings [23] and [7, 9] terminal weights can be taken into account. Theorem 3.2 and Proposition 3.3 remain valid for this setting. However, the interpretation of the suboptimality degrees α_N and $\widetilde{\alpha}$, respectively, via Estimate (2.7) does not remain valid since $V_N(\cdot)$ may not be monotone with respect to the prediction horizon N in this setting. Furthermore, note that the approach from [7, 9] is designed such that time varying control horizons are allowed which can lead to further sharpening the horizon estimates. This is particularly interesting, since the algorithmically based approach presented in [18] allows to carry out "classical" MPC safeguarded by enhanced stability estimates obtained for longer control horizons.

4. Numerical example. The proposed approach is applicable for systems governed by ordinary and partial differential equations, cf. [2]. Here, Condition (2.4) is numerically verified for the example of the synchronous generator given by

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= -b_1 x_3(t)\sin x_1(t) - b_2 x_2(t) + P\\ \dot{x}_3(t) &= b_3\cos x_1(t) - b_4 x_3(t) + E + u(t)\end{aligned} \tag{4.1}$$

with parameters $b_1 = 34.29$, $b_2 = 0.0$, $b_3 = 0.149$, $b_4 = 0.3341$, $P = 28.22$, and $E = 0.2405$, cf. [5]. Then, choosing a discretization parameter $T > 0$, the discrete time dynamics (2.1) may be defined by $f(x,u) = \Phi(T; x, \tilde{u}(\cdot))$ with $\tilde{u}(t) = u$ for all $t \in [0,T)$. $\Phi(T; x, \tilde{u}(\cdot))$ stands for the solution of the differential equation (4.1) at time T emanating from initial value x which is manipulated by the constant control function $\tilde{u}(\cdot)$. This construction represents a sampled-data system with zero order hold (ZOH) with sampling period T. For our numerical experiments T is set equal to 0.05.

Our goal is to stabilize this sampled-data system (4.1) at the equilibrium $x^\star \approx (1.124603730, 0, 0.9122974248)^T$. The running costs

$$\ell(x,u) = \int_0^T \|\Phi(t;x,\tilde{u}(\cdot)) - x^\star\|^2 + \lambda\|\tilde{u}(t)\|^2 \, dt = \int_0^T \|\Phi(t;x,\tilde{u}(\cdot)) - x^\star\|^2 \, dt + \lambda T\|u\|^2$$

are used where $\|\cdot\|$ denotes the Euclidean norm on $\mathbb{R}^3$ and $\mathbb{R}$, respectively. For the considered example the physically motivated state constraints $0 \leq x_1 < \pi/2$ and $x_3 \geq 0$ have to be taken into account. To this end, $\mathbb{X}$ is chosen to be a level set of $V_6(\cdot)$ which is located in the interior of the cube $[x_1^\star - 0.25, x_1^\star + 0.25] \times [-1,1] \times [x_3^\star - 0.75, x_3^\star + 0.75]$ and is control invariant according to our numerical experiments. This set is discretized with accuracy $\Delta x_i = 0.05$, $i \in \{1,2,3\}$, in each coordinate direction and consists, thus, of 3091 points, cf. [24, Subsection 4.4.1] for details.

Our first goal in this section is to determine a sequence $(c_n)_{n\in\mathbb{N}_0}$ satisfying Assumption 2.1. To this end, we compute, for each $\bar{x} \in \mathbb{X}$, an admissible control sequence $u_{\bar{x}}$ and define $c_n(\bar{x})$ by

$$\ell(x_{u_{x_0}}(n;x_0), u_{x_0}(n)) = c_n(x_0)\ell^\star(x_0),$$

cf. [24, Subsection 5.4.2] for details. We point out that this sequence does not have to be optimal which, on the one hand, eases the computations to be carried out but, on the other hand, may also lead to more conservative horizon estimates. In order to construct a suitable sequence which satisfies Inequality (2.4) independently of $\bar{x}$, the supremum has to be taken, i.e. $c_n := \sup_{\bar{x}\in\mathbb{X}} c_n(\bar{x})$. Hence, the γ_i, $i = 2,3,\ldots,N$, involved in Theorem 2.2 are given by $\gamma_i = \sum_{n=0}^{i-1} \sup_{\bar{x}\in\mathbb{X}} c_n(\bar{x})$. On the contrary, tighter bounds can be deduced by using Assumption 3.1 instead:

$$M_i := \sup_{x_0\in\mathbb{X}} \sum_{n=0}^{i-1} c_n(x_0) \leq \sum_{n=0}^{i-1} \sup_{x_0\in\mathbb{X}} c_n(x_0) = \gamma_i.$$

The estimates on the minimal stabilizing horizon decreases by 9 from $N = 41$ to $N = 32$, cf. Figure 4.1. Similar results are obtained for performance bounds $\overline{\alpha} > 0$.

Applying (3.2) with $\kappa \approx 1.29963597$ yields $N = 51$ as minimal stabilizing prediction horizon and requires, thus, an increment of 10 in contrast to the technique presented in [7, 9] and of 19 in comparison to the methodology introduced in this paper which consists of Assumption 3.1 from [23] in combination with Theorem 2.2 from [7, 9], cf. Figure 4.1. Clearly, directly determining the bounds γ_i, $i \in \{2,3,\ldots,N\}$, further improves the horizon estimates for this example. Here, however, we do not pursue this approach in order to indicate differences of the considered approaches.

Summarizing, deducing estimates based on Theorem 3.2 instead of Theorem 2.2 leads to a considerable reduction of the required prediction horizon length needed in order to guarantee asymptotic stability of the MPC closed loop. In conclusion, Assumption 3.1 allows to deduce each M_i separately for each index i. This additional flexibility can be exploited in order to derive better bounds and, thus, tighter horizon estimates in comparison to [7, 9]. A similar impact can be observed for many pde examples for which exponential controllability in terms of the stage cost is verified and, thus, constants $C \geq 1$ and $\sigma \in (0,1)$ are computed which typically depend on

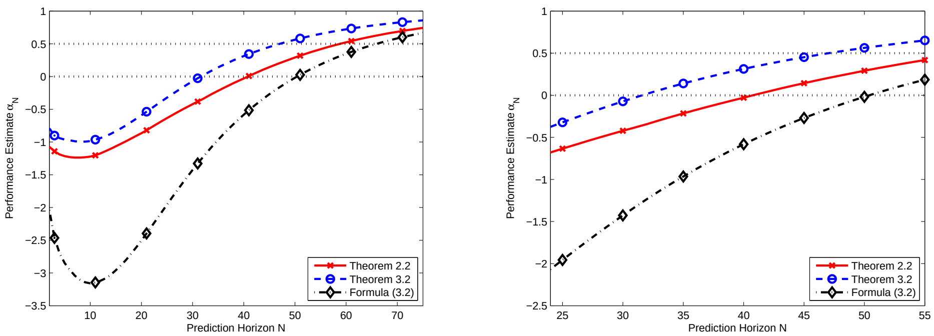

FIG. 4.1. *Comparison of the performance estimates resulting from Theorem 2.2, Theorem 3.2, and Formula 3.2.*

a parameter p. The presented methodology allows to individually optimize each γ_i, $i = 2, 3, \ldots, N$, with respect to this parameter instead of optimizing Formula (2.5), cf. [24, Chapter 5.4] for a pde example of a reaction diffusion equation.

5. Conclusions. We combined Assumption 3.1 from [23] with the technique proposed in [7, 9] in order to deduce tighter estimates on the required prediction horizon length in model predictive control without terminal constraints or costs. In addition, we showed that the assumption made in [7, 9] implies this assumption. Furthermore, we proved that the corresponding performance bound is tighter than its counterpart from 3.1 which was illustrated by a numerical example.

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