Visiting Whitney

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Abstract

Let $x_0 < x_1 < \cdots < x_{m-1}$, and set $I := [x_0.x_{m-1}]$ and $|I| := x_{m-1} - x_0$. Assume that for some $0 < \lambda \le 1$, we have $x_{j+1} - x_j \ge \lambda|I|$, for all $0 \le j \le m-2$. The Lagrange-Hermite polynomial of a function $f \in C^r(I)$, $L_{m-1}(x; f; x_0 \ldots, x_{m-1})$ is the unique polynomial of degree $m-1$, interpolating f at the points $x_0, \ldots, x_{m-1}$. For $m \ge \max\{r+1, 2\}$, the classical Whitney estimate of how well this polynomial approximates f in I is given by

$$|f(x) - L_{m-1}(x; f; x_0 \ldots, x_{m-1})| \le C(m, \lambda)|I|^r \omega_{m-r}(f^{(r)}, |I|, I), \quad x \in I,$$

where ω_k is that kth modulus of smoothness.

We allow some of the points to coalesce, specifically, we assume $x_0 \le x_1 \le \cdots \le x_{m-1}$ such that we only have $x_{j+r+1} - x_j \ge \lambda|I|$, for all $0 \le j \le m-r-2$. In other words, we assume that the Lagrange-Hermite polynomial interpolates f and its derivatives at x_j according to the multiplicity of the appearance of x_j in the the collection of points (but note that no x_j may appear more than $r+1$ times).We prove the above Whitney inequality for this situation.

Further, we will discuss an extension of Whitney's inequality to a local-type estimate.